CSE515: Algorithm Design
Notes on Lecture 21: Randomized Approximation Algorithm for MAX 3-SAT

Antoine Vigneron

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Proof that MAX 3-SAT is NP-hard. We use a reduction from 3-SAT. So suppose we have an algorithm A that solves MAX 3-SAT. Given k clauses $C_1, \ldots, C_k$, it returns the largest number M that can be satisfied. Then the following algorithm solves 3-SAT:

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procedure 3-SAT$(C_1, \ldots, C_k)$
    $M \leftarrow A(C_1, \ldots, C_k)$
    if $M = k$ **then**
        return SATISFIABLE
    return NOT SATISFIABLE

This is a polynomial-time reduction of 3-SAT to MAX 3-SAT, hence 3-SAT $\leqslant_p$ MAX 3-SAT. As 3-SAT is **NP**-hard, it implies that MAX 3-SAT is also **NP**-hard.